

UNIT-III : CALCULUS

CHAPTER

5

Term-I

CONTINUITY & DIFFERENTIABILITY

Syllabus

- Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.
- Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.



STAND ALONE MCQs

(1 Mark each)

Q. 1. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$ then which of the following can be a discontinuous function?

- (A) $f(x) + g(x)$ (B) $f(x) - g(x)$
(C) $f(x).g(x)$ (D) $\frac{g(x)}{f(x)}$

Ans. Option (D) is correct.

Explanation: Since $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$ are continuous functions, then by using the algebra of continuous functions, the functions $f(x) + g(x)$, $f(x) - g(x)$, $f(x).g(x)$ are also continuous functions but $\frac{g(x)}{f(x)}$ is discontinuous function at $x = 0$.

Q. 2. The function $f(x) = \frac{4-x^2}{4x-x^3}$

- (A) discontinuous at only one point
(B) discontinuous at exactly two points
(C) discontinuous at exactly three points
(D) none of these

Ans. Option (C) is correct.

Explanation: Given that,

$$f(x) = \frac{4-x^2}{4x-x^3},$$

then it is discontinuous if

$$\Rightarrow 4x - x^3 = 0$$

$$\Rightarrow x(4 - x^2) = 0$$

$$\Rightarrow x(2+x)(2-x) = 0$$

$$\Rightarrow x = 0, -2, 2$$

Thus, the given function is discontinuous at exactly three points.

Q. 3. The function $f(x) = \cot x$ is discontinuous on the set

- (A) $\{x = n\pi; n \in \mathbb{Z}\}$
(B) $\{x = 2n\pi; n \in \mathbb{Z}\}$
(C) $\left\{x = (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}\right\}$
(D) $\left\{x = \frac{n\pi}{2}; n \in \mathbb{Z}\right\}$

Ans. Option (A) is correct.



Explanation: Given that,

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

It is discontinuous at

$$\sin x = 0$$

$$\Rightarrow x = n\pi, n \in \mathbb{Z}$$

Thus, the given function is discontinuous at $\{x = n\pi : n \in \mathbb{Z}\}$.

Q. 4. If $f(x) = \begin{cases} mx+1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$, is continuous at $x = \frac{\pi}{2}$

then

(A) $m = 1, n = 0$ (B) $m = \frac{n\pi}{2} + 1$

(C) $n = \frac{m\pi}{2}$ (D) $m = n = \frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation: Given that,

$$f(x) = \begin{cases} mx+1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$$

is continuous function at $x = \frac{\pi}{2}$, then

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x)$$

$$\Rightarrow \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} + h\right)$$

$$\Rightarrow \lim_{h \rightarrow 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \rightarrow 0} \sin\left(\frac{\pi}{2} + h\right) + n$$

$$\Rightarrow \lim_{h \rightarrow 0} m\left(\frac{\pi}{2} - h\right) + 1 = \lim_{h \rightarrow 0} \cos h + n$$

$$\Rightarrow m\left(\frac{\pi}{2}\right) + 1 = 1 + n$$

$$\Rightarrow n = \frac{m\pi}{2}$$

Q. 5. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

(A) $25y$ (B) $5y$
(C) $-25y$ (D) $15y$

[CBSE Delhi Set-I 2020]

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} y &= Ae^{5x} + Be^{-5x} \\ \Rightarrow \frac{dy}{dx} &= 5Ae^{5x} - 5Be^{-5x} \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{d^2y}{dx^2} &= 25Ae^{5x} + 25Be^{-5x} \\ &= 25y \end{aligned}$$

Q. 6. If $y = \log_e \left(\frac{x^2}{e^2} \right)$, then $\frac{d^2y}{dx^2}$ equals

(A) $-\frac{1}{x}$ (B) $-\frac{1}{x^2}$
(C) $\frac{2}{x^2}$ (D) $-\frac{2}{x^2}$

[CBSE Delhi Set-III 2020]

Ans. Option (D) is correct.

Explanation:

$$\text{Given, } y = \log_e \left(\frac{x^2}{e^2} \right)$$

$$\Rightarrow y = 2\log_e x - \log_e e^2$$

$$\Rightarrow y = 2\log_e x - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{x^2}$$

Q. 7. The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is

(A) \mathbb{R} (B) $\mathbb{R} - \left\{ \frac{1}{2} \right\}$
(C) $(0, \infty)$ (D) none of these

Ans. Option (B) is correct.

Explanation: Given that,

$$f(x) = |2x - 1| \sin x$$

The function $\sin x$ is differentiable.

The function $|2x - 1|$ is differentiable, except

$$2x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}$$

Thus, the given function is differentiable $\mathbb{R} - \left\{ \frac{1}{2} \right\}$.

Q. 8. The function $f(x) = e^{|x|}$ is

(A) continuous everywhere but not differentiable at $x = 0$
(B) continuous and differentiable everywhere
(C) not continuous at $x = 0$
(D) none of these

Ans. Option (A) is correct.

Explanation: Given that,

$$f(x) = e^{|x|}$$

The functions e^x and $|x|$ are continuous functions for all real value of x .

Since e^x is differentiable everywhere but $|x|$ is non-differentiable at $x = 0$.

Thus, the given functions $f(x) = e^{|x|}$ is continuous everywhere but not differentiable at $x = 0$.

Q. 9. Let $f(x) = |\sin x|$, then

- (A) f is everywhere differentiable
- (B) f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbb{Z}$.
- (C) f is everywhere continuous but not differentiable at $x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$.
- (D) none of these

Ans. Option (B) is correct.

Explanation: Given that,

$$f(x) = |\sin x|$$

The functions $|x|$ and $\sin x$ are continuous function for all real value of x .

Thus, the function $f(x) = |\sin x|$ is continuous function everywhere.

Now, $|x|$ is non-differentiable function at $x = 0$.

Since $f(x) = |\sin x|$ is non-differentiable function at $\sin x = 0$

Thus, f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbb{Z}$.

Q. 10. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{4x^3}{1-x^4}$
- (B) $\frac{-4x}{1-x^4}$
- (C) $\frac{1}{4-x^4}$
- (D) $\frac{-4x^3}{1-x^4}$

Ans. Option (B) is correct.

Explanation: Given that,

$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow y = \log(1-x^2) - \log(1+x^2).$$

Differentiate with respect to x , we have

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}[\log(1-x^2)] - \frac{d}{dx}[\log(1+x^2)] \\ &= \frac{-2x}{1-x^2} - \frac{2x}{1+x^2} \\ &= -2x \left(\frac{2}{(1-x^2)(1+x^2)} \right) \\ &= -\frac{4x}{1-x^4} \end{aligned}$$

Q. 11. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{\cos x}{2y-1}$
- (B) $\frac{\cos x}{1-2y}$
- (C) $\frac{\sin x}{1-2y}$
- (D) $\frac{\sin x}{2y-1}$

Ans. Option (A) is correct.

Explanation: Given that,

$$y = \sqrt{\sin x + y}$$

$$\Rightarrow y^2 = \sin x + y$$

Differentiate with respect to x , we have

$$\Rightarrow 2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow (2y-1) \frac{dy}{dx} = \cos x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Q. 12. The derivative of $\cos^{-1}(2x^2-1)$ w.r.t. $\cos^{-1}x$ is

- (A) 2
- (B) $\frac{-1}{2\sqrt{1-x^2}}$
- (C) $\frac{2}{x}$
- (D) $1-x^2$

Ans. Option (A) is correct.

Explanation: Let

$$u = \cos^{-1}(2x^2-1)$$

$$\Rightarrow \frac{du}{dx} = -\frac{4x}{\sqrt{1-(2x^2-1)^2}}$$

$$\Rightarrow \frac{du}{dx} = -\frac{4x}{\sqrt{1-4x^4+4x^2-1}}$$

$$\Rightarrow \frac{du}{dx} = -\frac{4x}{\sqrt{-4x^4+4x^2}}$$

$$\Rightarrow \frac{du}{dx} = -\frac{2}{\sqrt{1-x^2}}$$

And, $v = \cos^{-1}x$

$$\frac{dv}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\text{Thus, } \frac{du}{dv} = 2$$

Q. 13. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is

- (A) $\frac{3}{2}$
- (B) $\frac{3}{4t}$
- (C) $\frac{3}{2t}$
- (D) $\frac{3}{4}$

Ans. Option (A) is correct.

Explanation: Given that,

$$x = t^2 \text{ and } y = t^3$$

$$\text{Then, } \frac{dx}{dt} = 2t \text{ and } \frac{dy}{dt} = 3t^2$$

Thus,

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2}$$



ASSERTION AND REASON BASED MCQs

(1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. Assertion (A): $|\sin x|$ is continuous for all $x \in R$.

Reason (R): $\sin x$ and $|x|$ are continuous in R .

Ans. Option (A) is correct.

Explanation: $\sin x$ and $|x|$ are continuous in R . hence R is true.

Consider the functions $f(x) = \sin x$ and $g(x) = |x|$ both of which are continuous in R .

$$g \circ f(x) = g(f(x)) = g(\sin x) = |\sin x|.$$

Since $f(x)$ and $g(x)$ are continuous in R , $g \circ f(x)$ is also continuous in R .

Hence A is true.

R is the correct explanation of A.

Q. 2. Assertion (A): $f(x) = \tan^2 x$ is continuous at $x = \frac{\pi}{2}$.

Reason (R): $g(x) = x^2$ is continuous at $x = \frac{\pi}{2}$.

Ans. Option (D) is correct.

Explanation: $g(x) = x^2$ is a polynomial function. It is continuous for all $x \in R$.

Hence R is true.

$f(x) = \tan^2 x$ is not defined when $x = \frac{\pi}{2}$.

Therefore $f\left(\frac{\pi}{2}\right)$ does not exist and hence $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

A is false.

Q. 3. Consider the function $f(x) = \begin{cases} kx, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$

which is continuous at $x = 0$.

Assertion (A): The value of k is -3 .

Reason (R): $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Ans. Option (A) is correct.

Explanation:

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

This is the definition for modulus function and hence true.

Hence R is true.

Since f is continuous at $x = 0$,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

Here $f(0) = 3$,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \frac{kx}{|x|} = \lim_{x \rightarrow 0^-} \frac{kx}{-x} = -k$$

$$\therefore -k = 3 \text{ or } k = -3.$$

Hence A is true.

R is the correct explanation of A.

Q. 4. Consider the function

$$f(x) = \begin{cases} x^2 + 3x - 10, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$

which is continuous at $x = 2$.

Assertion (A): The value of k is 0.

Reason (R): $f(x)$ is continuous at $x = a$, if

$$\lim_{x \rightarrow a} f(x) = f(a).$$

Ans. Option (D) is correct.

Explanation:

$f(x)$ is continuous at $x = a$, if $\lim_{x \rightarrow a} f(x) = f(a)$.

\therefore R is true.

$$\lim_{x \rightarrow 2} f(x) = f(2) = k$$

$$\lim_{x \rightarrow 2} \frac{(x+5)(x-2)}{x-2} = k$$

$$\therefore k = 7$$

Hence A is false.

Q. 5. Assertion (A): $|\sin x|$ is continuous at $x = 0$.

Reason (R): $|\sin x|$ is differentiable at $x = 0$.

Ans. Option (C) is correct.



Explanation: Since $\sin x$ and $|x|$ are continuous functions in R , $|\sin x|$ is continuous at $x = 0$.

Hence A is true.

$$|\sin x| = \begin{cases} -\sin x, & \text{if } x < 0 \\ \sin x, & \text{if } x \geq 0 \end{cases}$$

$$f(0) = |\sin 0| = 0$$

$$\text{LHD} = f'(0^-) = \lim_{x \rightarrow 0^-} \frac{-\sin x - 0}{x} = -1$$

$$\text{RHD} = f'(0^+) = \lim_{x \rightarrow 0^+} \frac{\sin x - 0}{x} = 1$$

At $x = 0$, $\text{LHD} \neq \text{RHD}$.

So $f(x)$ is not differentiable at $x = 0$.

Hence R is false.

Q. 6. Assertion (A): $f(x) = [x]$ is not differentiable at $x = 2$.

Reason (R): $f(x) = [x]$ is not continuous at $x = 2$.

Ans. Option (A) is correct.

Explanation: $f(x) = [x]$ is not continuous when x is an integer.

So $f(x)$ is not continuous at $x = 2$. Hence R is true.

A differentiable function is always continuous.

Since $f(x) = [x]$ is not continuous at $x = 2$, it is also not differentiable at $x = 2$.

Hence A is true.

R is the correct explanation of A.

Q. 7. Assertion (A): A continuous function is always differentiable.

Reason (R): A differentiable function is always continuous.

Ans. Option (D) is correct.

Explanation: The function $f(x)$ is differentiable at $x = a$, if it is continuous at $x = a$ and

$$\text{LHD} = \text{RHD at } x = a.$$

A differentiable function is always continuous.

Hence R is true.

A continuous function need not be always differentiable.

For example, $|x|$ is continuous at $x = 0$, but not differentiable at $x = 0$.

Hence A is false.

Q. 8. Assertion (A): If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, then

$$\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

Reason (R): $\sin^{-1}(6x\sqrt{1-9x^2}) = 3\sin^{-1}(2x)$

Ans. Option (C) is correct.

Explanation:

$$\text{put } 3x = \sin \theta \text{ or } \theta = \sin^{-1} 3x$$

$$y = \sin^{-1}(6x\sqrt{1-9x^2}) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$= 2\sin^{-1} 3x$$

$$\therefore \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

A is true. R is false.



CASE-BASED MCQs

Attempt any four sub-parts from each question.

Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Ms. Remka of city school is teaching chain rule to her students with the help of a flow-chart

The chain rule says that if h and g are functions and $f(x) = g(h(x))$, then

$$f'(x) = (g(h(x)))' = g'(h(x)) h'(x)$$

- keep the inside
- take derivative
of outside

by derivative
of the inside

Let $f(x) = \sin x$ and $g(x) = x^3$

Q. 1. $f \circ g(x) =$ _____.

(A) $\sin x^3$

(B) $\sin^3 x$

(C) $\sin 3x$

(D) $3\sin x$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(x^3) \\ &= \sin(x^3) \end{aligned}$$

Q. 2. $g \circ f(x) =$ _____.

(A) $\sin x^3$

(B) $\sin^3 x$

(C) $\sin 3x$

(D) $3\sin x$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(\sin x) \\ &= \sin^3 x \end{aligned}$$



Q. 3. $\frac{d}{dx}(\sin^3 x) = \underline{\hspace{2cm}}$.

- (A) $\cos^3 x$ (B) $3\sin x \cos x$
(C) $3\sin^2 x \cos x$ (D) $-\cos^3 x$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\frac{d}{dx}(\sin^3 x) &= 3\sin^2 x \frac{d}{dx}(\sin x) \\ &= 3\sin^2 x \cos x\end{aligned}$$

Q. 4. $\frac{d}{dx} \sin x^3 = \underline{\hspace{2cm}}$.

- (A) $\cos(x^3)$ (B) $-\cos(x^3)$
(C) $3x^2 \sin(x^3)$ (D) $3x^2 \cos(x^3)$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned}\frac{d}{dx}(\sin x^3) &= \cos x^3 \frac{d}{dx}(x^3) \\ &= 3x^2 \cos x^3\end{aligned}$$

Q. 5. $\frac{d}{dx}(\sin 2x)$ at $x = \frac{\pi}{2}$ is $\underline{\hspace{2cm}}$.

- (A) 0 (B) 1
(C) 2 (D) -2

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned}\frac{d}{dx}(\sin 2x) &= \cos 2x \frac{d}{dx}(2x) \\ &= 2\cos 2x \\ \left. \frac{d}{dx}(\sin 2x) \right|_{x=\frac{\pi}{2}} &= 2\cos 2 \times \frac{\pi}{2} = 2\cos \pi \\ &= 2(-1) \\ &= -2\end{aligned}$$

II. Read the following text and answer the following questions on the basis of the same:

A potter made a mud vessel, where the shape of the pot is based on $f(x) = |x - 3| + |x - 2|$, where $f(x)$ represents the height of the pot.



[CBSE QB 2021]

Q. 1. When $x > 4$ what will be the height in terms of x ?

- (A) $x - 2$ (B) $x - 3$
(C) $2x - 5$ (D) $5 - 2x$

Ans. Option (C) is correct.

Explanation: The given function can be written as

$$f(x) = \begin{cases} 5 - 2x, & \text{if } x < 2 \\ 1, & \text{if } 2 \leq x < 3 \\ 2x - 5, & \text{if } x \geq 3 \end{cases}$$

When $x > 4$, $f(x) = 2x - 5$

Q. 2. Will the slope vary with x value?

- (A) Yes (B) No
(C) Can't say (D) In complete data

Ans. Option (A) is correct.

Explanation:

$$f'(x) = \begin{cases} -2, & \text{if } x < 2 \\ 0, & \text{if } 2 \leq x < 3 \\ 2, & \text{if } x \geq 3 \end{cases}$$

Q. 3. What is $\frac{dy}{dx}$ at $x = 3$

- (A) 2
(B) -2
(C) Function is not differentiable
(D) 1

Ans. Option (C) is correct.

Explanation: $f(x)$ is not differentiable at $x = 2$ and $x = 3$.

Q. 4. When the value of x lies between (2, 3) then the function is

- (A) $2x - 5$ (B) $5 - 2x$
(C) 1 (D) 5

Ans. Option (C) is correct.

Explanation: In (2, 3), $f(x) = 1$

Q. 5. If the potter is trying to make a pot using the function $f(x) = [x]$, will he get a pot or not? Why?

- (A) Yes, because it is a continuous function
(B) Yes, because it is not continuous
(C) No, because it is a continuous function
(D) No, because it is not continuous

Ans. Option (D) is correct.

Explanation: $[x]$ is not continuous at integral values of x .

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